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On Empirically Equivalent Systems of the World with Conflicting Ontologism: Three Case-Studies

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Abstract

Quine's under-determination thesis has various formulations. One of them states that, for any theory formulation, there is another formulation that is empirically equivalent to it but logically incompatible with it, and cannot be rendered logically equivalent to it by any re-construal of predicates. This definition is coherent with the following simpler one: we may face several theories (possibly not only two of them) which are empirically equivalent but conflicting. I introduce and discuss a variant in which a possible way for two theories to conflict is to exhibit different ontologies (say, furniture's of the world). This defines what I call the ontological under-determination (possibly with other terminological variants). In this paper, the thesis of ontological under-determination is discussed by relying on three examples pertaining to physical sciences (i)one pertaining to classical mechanics : Newton's formulation of classical mechanics versus Hamilton-Jacobi's formulation (ii) causal interpretations of quantum mechanics (mainly : pilot wave theory) versus Copenhagen interpretation and (iii) Mie's theory versus Lorenz' theory, describing the electromagnetic interaction between an illuminating plane wave and a scattering sphere defined by its complex refractive index and by its diameter. Discriminations between ontological under-determined theories are afterward achieved by using implicative arguments.

Introduction

Many scientists have opted for a strong (sometimes also called naive) realist interpretation of science. Such a realist vision has been challenged by many philosophical doctrines, since a long time, such as by various kinds of idealisms, obviously including the transcendental idealism of Kant. More recently, the realistic interpretation has been challenged by quantum mechanics too.

In this paper, I discuss another source of troubles for those who would like to share a realist interpretation of science, namely Quince's thesis (or Duhem-Quine's thesis) of under-determination of theories by experiments (or better said, a variant of it defined later). Quince's under-determination thesis (or in short Quince's under-determination) may be given various formulations. One of them states that, for any theory formulation, there is another formulation that is empirically equivalent to it but logically incompatible with it, and cannot be rendered logically equivalent to it by any re-construal of predicates. This is a rather abstract enunciation and the reader is kindly requested to refer to [1] for a more thorough discussion. The previous abstract definition is however coherent with the following simpler one we may face several theories (possibly not only two of them) which are empirically equivalent but conflicting. There are different ways for theories to conflict. One of them is indeed on the stage when we are facing two logically incompatible theories.

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However, I shall consider another possible way to encounter conflicting theories, namely: theories do conflict when they exhibit different ontologies (say, furniture's of the world). This means that, under such circumstances, if we were demanding that science had to answer the question to know which are the entities populating the world, and then we would be definitely puzzled. And we should have to adopt a strict positivist posture, being content to answer the question: how?, without any attempt to answer the questions : what?, or still harder to please : why?

The thesis to consider is then the one stating that theories are ontologically under-determined by experiments (or, if you prefer to avoid taking any risk: theories may be ontologically under-determined by experiments). This may be viewed as a variant of Quince's under-determination which we may however prefer to discuss in its own right, neither without any more referring touché nor to Quince. For this last reason, I shall refer to this thesis as the thesis of ontological under-determination (possibly with other terminological variants). Ontological under-determination is easier to grasp than Quince's under-determination which remains, in the very words of Quine himself, vague and modest [1]. In particular, while Quine'sunder-determination thesis has not been satisfactorily exemplified (at least, in face of the eyes of physicists), I may provide quite decent examples for the ontological under-determination thesis.

More explicitly, in this paper, I shall discuss three examples, orcas-studies, relevant to physical sciences (i) one pertaining to classical mechanics : Newton's formulation of classical mechanics versus Hamilton-Jacobi's formulation (ii) causal interpretations of quantum mechanics (mainly : pilot wave theory) versus usual (Copenhagen)interpretation and (iii) Mie's theory versus Lorenz' theory, describing the electromagnetic interaction between an illuminating plane wave and a scattering sphere defined by its complex refractive index and by its diameter. An important issue is that discriminations between undecidable ontologically under-determined theories are afterward achieved by using ampliative arguments, that is to say arguments allowing one to decide between undesirables.

The paper is organized as follows. In section II, Quine's under-determination thesis and ontological under-determination thesis are more extensively exposed. Section III expounds the first case-study, in the framework of classical mechanics. Section IV expounds the second case-study, in the framework of quantum mechanics. Section V revisits under-determinations from the point of view of undesirability, and in particular introduces the concept of implicative arguments aiming to the discrimination between conflicting empirically equivalent theories. Section introduces a non-singularity principle which is afterward used to decide between undesirables for the first and second case-studies. Section VII is devoted to a third case-study, pertaining to electromagnetism. Section Ivies a conclusion.

Quince's and ontological under-determinations

Forgetting precursors (to save room), Quince's under-determination is discussed in "Word and Object," regarded as the most important book beguine [2] or in the "Pursuit of Truth" [3], his last book. The French reading reader may also find a synthetic account of the work of Quince in Ref. [4]. But a specific and specialized report is available in a famous paper by Quince [1] to which I shall better refer.

At the beginning of his paper, Quince stated what can be taken as the essence of the under-determination thesis, at least as a starting point to be refined, as follows: *If all observable events can be accounted florin one comprehensive scientific theory- one system of the world, to echo Duhem's echo of Newton- then we may expect that they can all be accounted for equally in another, conflicting, system of the world*. Such an expectation may rely on the examination of *how scientists work. For they do not rest with mere inductive generalizations of their observations: mere extrapolation to observable events from similar observed events. Scientists invent hypotheses that talk of things beyond the reach of observation. These hypotheses are related to observation only by a kind of one-way implication, namely, the events we observe are what a belief in the hypotheses would have led us to expect. These observable consequences of the hypotheses do not, conversely, imply the hypotheses.* In the words of Quine, this defines a doctrine, a doctrine saying that *natural science is empirically under-determined,* a doctrine from which we can say that it *is plausible insofar as it is intelligible, but it is less readily intelligible that it may seem*. The doctrine is afterward exposed in general terms (i.e. again without any convincing example) which alone do not allow one to be clearly convinced, something which is implicitly acknowledged by Quine when, instead of using the word "truthfulness," he is content of using the word "plausibility".

Redefining more carefully the thesis, Quine arrived to the following formulation: under-*determination says that for anyone theory formulation there is another that is empirically equivalent to it but logically incompatible with it, and cannot be rendered logically equivalent to it by any re-construal of predicates*.

Eventually, after more discussions, he landed on the following form, that *our system of the world is bound to have empirically equivalent alternatives which, if we were to discover them, we would see no way of reconciling by re-construal of predicates.* Whether the thesis, under this *vague and modest* form, is true or not, this remains for Quine *an open question*. However, Quine, as he said, believed to it. A logician might roar in face of such an incongruous object: a thesis which cannot be demonstrated but must be a matter of faith. At this stage, we may certainly agree with Harré [5] when he spoke of the *myth of the under determination of theories by data*.

The most difficult thing to correctly state Quine's under-determination thesis is likely to precisely define what a theory is and/or to define to which kinds of theories the thesis applies. Consider for instance the old theory of Bohr correctly explaining the energy levels of the hydrogen atom. The energy levels predicted by Bohr are in perfect agreement with the energy levels predicted by Schrödinger from his equation. Should we consider that we have here a simple example of the application of Quine's under-determination thesis? Certainly not, and this for two reasons. First, Bohr's theory is more a model than a theory (Bohr himself was aware of the fact). Second, not all predictions of Bohr's model agree with all predictions of Schrödinger's wave mechanics. Therefore, we here have two theories (let us accept the name of theory) which are not experimentally equivalent *in all aspects.*

In the words of Bricmont [6], indeed, the question, regarding this issue, is to know what counts as a theory. For instance, according to Bricmont[6], saying that a certain disease is caused by a virus presumably counts as a theory. We might then, in the same mood, propose other examples, such as: "saying that the Earth is flat" presumably counts as a theory, "saying that I can see cows in the field means that there are cows in the field" presumably counts as a theory, or" saying that when the sun goes to sleep, it turns to a cream cheese "presumably counts as theory. The last example is an allusion to a quotation from Squires to be served later. Therefore, actually, we would like to know what is the minimal level of complexity required for a formal statement, or for a set of formal statements, to decide whether it deserves, or not, the honorific designation of : theory.

Quine himself was aware of this difficulty. On one hand, one of the formulation of the thesis, the one I provided above at the beginning of this subsection, refers to *all observable events*. If we interpret this expression as concerning what it literally means, i.e. all observable events, whether they have been already actually observed, in the past, or are to be potentially observed, in the future, then the under-determination thesis should be applied to the whole of science, however not yet completed(see [7] including the introduction). The validity of the thesis when applied to the whole of science is certainly something overwhelmingly difficult to convincingly establish, if only at least for a simple reason: we do not yet possess even one theory of everything, encompassing the whole of science. On the other hand, the thesis is of no value for weak theories, i.e. *theories that imply no rich store of observation conditionals,* cases where the thesis can be trivially and therefore uninterestingly satisfied. So, at the best of our present common understanding, examples of Quine's under-determination thesis should be searched for theories which are rich enough without being, because unrealistic, the whole of science.

Another complementary point of view may be gained if we attach a domain of validity (or of application) to each theory, from the simple ones (fire burns, water is humid) to the most achieved one, the *ultimate theory* which would generate the whole of science, encompassing all domains of validity, if any. If we affirm, by a *fiat*, that we are just interested with energy levels of the hydrogen atom, therefore defining in this way a domain of validity, then we could take the agreement between Bohr's model and the results from Schrödinger's equation as a decent example of Quine's under-determination. This, however, is something that we are very reluctant to accept, because the levels of the hydrogen atom define a very weak domain of application. Therefore, the problem is not only to define exactly what can be named a theory, but also what are admissible domains of validity (or of application). There is indeed a quasi-continuum of theoretical constructions, from a simple experimental law to a consistent theory like relativity, and there is also a quasi-continuum of domains of validity, from a certain domain, like the quantum mechanical domain, to sub-domains, and sub-sub-domains, like the one concerning only the predictions of energy levels of the hydrogen atom.

A pertinent analysis of Quine's anthropology is available from Laugier-Rabaté[8]. According to this author, it appears that Quine did not provide convincing examples for his thesis of the under-determination of the radical translation (another famous thesis from Quine, not discussed in this paper). In addition, furthermore, the lack of convincing example is also characterizing the thesis of the under-determination of theories by experiments. Quine himself would certainly agree with this statement if we refer to the mood expressed in his Erkentniss-paper [1].

Another author which is often quoted as relevant to the issue of Quine's under-determination is Van Fraassen, in particular in reference to his book" Laws and Symmetries" [9]. Van Fraassen distinguished between theoretical equivalence and empirical equivalence.

For him, if two statements are logically (theoretically) equivalent, then they are saying the same thing. However, if we consider a theory as a kind of statement, say a meta-statement, we have at least one example to demonstrate that the affirmation of Van Fraassen is erroneous. This example concerns the Newton's and Hamilton-Jacobi's formulations of classical mechanics (soon to be extensively discussed). Both theories may be viewed as logically, theoretically, equivalent by the simple fact that they are mathematically equivalent. However, they are not saying the same thing. Contrarily, they are proposing two drastically different visions of the world, one with local trajectories, the other with local trajectories embedded into a non-local field.

The status of empirical equivalence alone may however be simpler to discuss without any flaw. We just have to admit the possibility of theories saying the same thing, not on the nature of the world, but on the predictions of experimental facts. But the example of Newton's and Hamilton-Jacobi's formulations demonstrates that two theories which are logically equivalent, and saying the same thing as far as empirical facts are concerned, may be deeply contradictory if we interpret them in a larger framework, a framework in which we are concerned with the visions of the world they provide. Let us furthermore note that, in his book [9], Van Fraassen discussed the issue in general terms, without, as Quine, referring to explicit concrete examples. This is unfortunate because well designed examples may be illuminating, and physicists would certainly not be satisfied if specific examples, providing a support for understanding and discussion, are not explicitly put forward.

Elsewhere, Van Fraassen[10] discussed the lack of uniqueness asserted by Quine's under-determination in association with the existence of different interpretations of quantum mechanics. As he wrote, *why then be interested in interpretation at all? If we are not interested in the metaphysical question of what the world is really, like, what need is there to look into these issues? Well, we should still be interested in the question of how the world could be the way quantum mechanics - in its metaphysical vagueness but empirical audacity-says it is. That is the real question of understanding. To understand a scientific theory, we need to see how the world could be the way that the theory says it is. An interpretation tells us that.* Later on, he added: *the answer is not unique, because the question 'How could the world be the way the theory says it is?' is not the sort of question to call for a unique answer*.

The issue of interpretation is also considered by Cushing [11]: *Very loosely, the formalism refers to the equations and calculation rules that prove empirically adequate (i.e. getting the numbers right) and the interpretation refers to the accompanying representation the theory gives us about the physical universe (i.e. the picture story that goes with the equations of what our theory "really" tells us about the world). Since a (successful) formalism does not uniquely determine its interpretation, there may be two radically different interpretations (and ontologies) corresponding equally well to one adequate formalism. This can be taken as an instantiation of the Duhem-Quinethesis of indetermination of theory by an empirical base. Even if one wants to restrict (and, arguably, that would be a mistake) the Duhem-Quine thesisto different formalisms each handling equally well a given body of empirical information, there nevertheless remains the interesting and important point of opposing ontologies equally well supported by a common empirical base.*

The correct understanding of this Cushing's quotation requires us to tell more on the terminologies we may use. I have extensively discussed Quine's thesis of under-determination. As underlined by Quine himself [1], this thesis *is not to be confused with holism. It isholism that has rightly been called the Duhem thesis and also, rather generously, the Duhem-Quine thesis. It says that scientific statements arenot separately vulnerable to adverse observations, because it is only jointly as a theory that they imply their observable consequences.*

In contradiction with this quotation from Quine, but in agreement with other authors, Cushing used the terminology "Duhem-Quine" to designate Quine's under-determination. The association of the name of Duhem with Quine's under-determination thesis is not undeserved [12]. Sometimes, the expression of Duhem-Quine theorem (a philosophical theorem not demonstrated!) may be found to express an enunciation of Quine's under-determination thesis.

Furthermore, and mostly importantly, both van Fraassen and Cushing, who drive us toward the issues of interpretations and ontologies, allow one to put forward another point of view on the interpretation of Quine's under-determination thesis, a point of view concerning ontologies associated with theories and interpretations of theories. Let us enunciate an under-determination thesis under the following form: we may have several conflicting theories which are empirically equivalent. Now, such annunciation is not complete if we do not specify the meaning of the word "conflicting". The meaning could be as given by Quine, for example: two theories are conflicting when they are logically incompatible, and cannot be rendered logically equivalent by any re-construal of predicates. From now on, I use another definition: two theories are conflicting when they exhibit different ontologies. The thesis of ontological under-determination then states that we may have empirically equivalent theories which do not exhibit the same ontologies. It remains to define what ontology is: the ontology of a theory tells us which kinds of entities are populating the world. It describes the *furniture of the world* as implied by the interpretation of the theory. For instance, the furniture of classical physics is made out from localized objects and extended fields (possibly taking the form of waves). From this point of view, ontological under-determination may be viewed as a sub-thesis of Quine's thesis which, as mentioned in the introduction, may also be discussed in its own right, and which, from now on, may be discussed without referring any more to Duhem, Quine, or van Fraassen. Nevertheless, as a last remark, and as a corollary, let us mention that the experimental truth depends immanently on the conceptual scheme of our language and on the entities that this language allows us to manipulate (this insistence on the fact that we have to use a language, with its inherent limitations, is also typical of Wittgenstein [4], [13]). In particular, it might happen that the ultimate furniture of the world (ontology) could be outside of any possible human experience.

We are now going to provide explicit and well-defined examples (lacking inVan Fraassen or in Quine) of under-determination, more specifically of ontological under-determination, relying on theories which are rich enough, although not describing the whole of science, namely classical mechanics, quantum mechanics, and electromagnetism.

First case-study: classical mechanics.

The possibility of an under-determination of theories by experiments is usually much disliked by physicists, and the vagueness which remains attached to this concept does not help. Squires [14] expressed this point with a bit of humor or even irony by saying that *the statement that two theories, both of which fit the data, are equally good can be seen to be unreasonable if we note that a theory in which the sun always turns into cream cheese as it disappears over the horizon, and turns back again later, gives a perfectly adequate account of my observation*. For sure, Squires should not be taken too seriously in this example concerning theories which are not rich enough. Indeed, to be convinced, the physicist demands convincing examples. This section introduces the first of them, concerning ontological under-determination with a rich enough domain of validity which is the one of (no relativistic) classical mechanics. We restrict ourselves to the mechanics of matter points, however without any loss of generality, insofar as an extended body is viewed as a collection of matter points (in the framework of classical mechanics).

It is known that classical mechanics can be declined under four different formulations, which are empirically equivalent. These are the Newton's, Lagrange's, Hamilton's, and Hamilton-Jacobi's formulations. We only need in this paper to discuss Newton's and Hamilton-Jacobi's formulations. Discussing Newton's formulation is fast. It is just sufficient (and necessary) to recall that, using the basic law telling us that force is equal to mass multiplied by acceleration, and integrating with initial conditions, we can build trajectories of matter points.

We however need to be a bit more eloquent with Hamilton-Jacobi's formulation, in particular because it is less familiar (see for instance Louis de Broglie [15], Blotkhintsev[16], Landau and Lifchitz[17], and Holland [18]).Hamilton-Jacobi's formulation of no relativistic classical mechanics of a matter point relies on an equation, that I shall call Hamilton-Jacobi's equation, reading as:

$$-\frac{\partial S}{\partial t}=\frac{1}{2m}\left(\frac{\partial S}{\partial x_j}\right)^2+V \qquad (1)$$

This equation allows one to study the motions of a particle of mass *m* in a potential $V=V(\mathrm{x}_j,\mathrm{t})$. The x_j's denote Cartesian coordinates and t is the time. The field $S=S\left(x_j,t\right)$ is a real field that I shall call the Jacobi's field. Eq. 1 has to be complemented by two other equations reading as:

$$W=-\frac{\partial S}{\partial t} \qquad (2)$$

$$p_j=\frac{\partial S}{\partial x_j} \qquad (3)$$

in which *W* is the energy and p_j is the momentum. From Eq. 2, we see that *S* is an action (energy multiplied by time) and, from now on, we may call it the action. Also, inserting Eqs. 2 and 3 in Eq. 1, we see that we obtain *W=T+V*, which should be enough to convince us of the mathematical (and empirical) equivalence between Newton's and Hamilton-Jacobi's formulations.

For a conservative motion, the energy (that we denote *E* in that case) is constant along each particular motion, and Eq. implies:

$$S\ (\mathrm{x}_j,\mathrm{t})=\mathrm{S}_0(x_j)-Et \qquad (4)$$

Inserting Eq. 4 into Eq. 1, we obtain:

$$\left(\frac{\partial S_0}{\partial x_j}\right)^2=2m(\mathrm{E}-\mathrm{V}) \qquad (5)$$

We now consider the locus of the points for which S_0 possesses a given value C_0:

$$S_0(\mathrm{x}_j)=C_0 \qquad (6)$$

Eq. 6 shows that the locus is a time-independent surface. There is one surface, and only one, containing a point *P* of space, according to $C_0=S_0(\mathrm{x}_j(\mathrm{P}))$. The whole space is therefore filled by a set of motionless surfaces forming what I call the Jacobi's static field. From Eqs. 3 and 4, we have:

$$p_j=\left(\frac{\partial S}{\partial x_j}\right)=\left(\frac{\partial S_0}{\partial x_j}\right) \qquad (7)$$

Therefore, p_j is the gradient of S (and S_0). This means that trajectories are orthogonal to the surfaces S (and to the surfaces S_0).Next, we consider the locus of the points for which the action S possesses a given value C:

$$S(x_j,t)=C \qquad (8)$$

Eq. 8 shows that the locus is still a surface but which now depends onetime. When times goes on, the surface moves and, in general, experiences deformation. For a given time t, the moving surface $S\left(x_j,t\right)=C$ coincides with a motionless surface $S_0\left(x_j\right)=C_0$, according to, from Eq.4 : $C=C_0-Et$. Therefore, when time goes on, the moving surface $S=C$ sweeps over all motionless surfaces $S_0=C_0$.

Hence, we possess two formulations of classical mechanics, the Newton's and Hamilton-Jacobi's formulations, which are mathematically and empirically equivalent, but which are in contradiction insofar as they do not say the same thing on the world, i.e. they do not exhibit the same ontologies. Indeed, Newton's formulation only deals with localized objects following trajectories, while Hamilton-Jacobi's formulation deals with localized objects following trajectories, dressed by an unobservable, non-local, action field filling the space. Because they are therefore conflicting, these two formulations must be counted as two different theories (not only as two different formulations of classical mechanics). They have to be taken as forming an example of ontological under-determination, for rich enough theories, excepted if we just shrug our shoulders and rashly sweep S under the carpet.

A possible escape could be as follows. Since we are used to observe trajectories of macroscopic objects in our everyday experience (macroscopic objects, not matter points however), we are keen to believe that the field S of the Hamilton-Jacobi's formulation does not have any physical reality, but is simply an intermediary tool for computations. After all, this field does not pertain to our sense data. Then, we would have found a way to discriminate between Newton's and Hamilton-Jacobi's formulations (theories)in favor of Newton's formulation, by denying that S could be an ontological entity. As we shall see in this paper, this point of view, although highly reasonable, will reveal itself to be erroneous, because S is an anticipation of the phase of the wave-function Ψ (more generally: state vector) of quantum mechanics, see Eq. 22, which, although unobservable, is the most fundamental kind of entity populating the quantum world. A much more extended discussion of this issue (relationship between S and Ψ) is available from [19].

As another comment, I would like to mention that one colleague of mine denies the fact that Newton's and Hamilton-Jacobi's formulations exhibit different ontologies. He would prefer to say that the ontology of Hamilton-Jacobi's formulation is simply richer than that of Newton's formulation. To oppose to this opinion, let us consider the metaphor according to which ontology presents the furniture of the world, and extend it. Let us then consider a room (the world of Newton) in which there is one table, and another room (another world, the one of Hamilton-Jacobi)in which there is one table and four chairs. Surely, as a certain point of view, we may state that the second world is richer than the first one. Surely also, they are deeply different: definitely, they are not the same worlds (one allows us to sit comfortably, the other not). The second point of view is therefore the one which is taken in this paper. As we shall see, the two worlds are actually so different that the so-called richer world will provide an easy access to quantum mechanics, which deal with particles and fields, in contrast with the other one which is so poor that nothing more can be done with it.

Second case-study: quantum mechanics.

For this second case-study, we are going to compare two different kinds of interpretation (actually two different theories) of quantum mechanics, the usual one, the one of the text-books, which is conventional, standard, orthodox... and the causal theories, the heretical and unorthodox ones, mainly developed by Louis de Broglie (double solution, and pilot wave) and David Bohm (pilot wave). The usual quantum mechanics, on one hand, exhibits an intrinsic indeterminacy, associated with the Von Neumann's projection postulate governing any quantum measurement process. The causal theories, on the other hand, restore the determinism in quantum mechanics by invoking a sub-quantum level containing Newton-like trajectories of hidden particles. These causal theories are examples of what is called "hidden variables theories." They are constructed under the constraint that they should produce the same predictions as quantum mechanics.

If quantum mechanics were, one day or another day, to be falsified by experiments (in the sense of Popper), then the causal theories, because they produce the same predictions as quantum mechanics, would be simultaneously falsified too. Therefore, the causal theories and quantum mechanics are empirically equivalent. They however produce different ontologism, one which is deterministic with hidden Newton-like trajectories, the other one which is in deterministic without any trajectory.

A brief history of causal theories may be built from references by the two main protagonists of the enterprise, namely Louis de Broglie and Bohm, e.g. [15], [20], [21], [22], [23], [24], [25], [26], [27] or [28], for a selection. Complementary historical analysis and reports are also available from Vigier[29], Jammer [30], [31], Pinch [32], Wheeler and Zurek[33], Holland [18], or Cushing [34], among others. A more general point of view, not only on causal theories, but on larger classes of hidden variables theories, is also discussed by Freistadt[35], Belinfante[36], Pipkin[37] or d'Espagnat[38]. More generally, let us note that there is also a vast and sometimes recent literature on the unorthodox Bohmian interpretation of quantum mechanics versus the orthodox Copenhagen one. This literature is comprehensively available from [19] containing more than 500 references.

I shall be content with a discussion of the pilot wave theory as developed by Bohm in two famous papers dated 1952 [26], [39].To begin with, Bohm accepted the validity of Schrödinger's equation to describe the wave function Ψ of quantum mechanics. However, Ψ is viewed as an objective field, exactly such as an electromagnetic field satisfying Maxwell's equations. We do not know from first principles why there should be an electromagnetic field, nor do we know *what an electromagnetic field is*. Therefore, why should we not adopt the same complacency for Ψ than the one we have for E and H, or for their unification in an electromagnetic tensor H_{ij}? In the words of Bohm : *in the last analysis, there is, of course, no reason why a particle should not be acted on by a Ψ -field, a gravitational field, a set of meson fields, and perhaps by still other fields that have not yet been discovered.*

Formally, we start from Schrödinger's equation:

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x_j{}^2}+V\Psi \qquad (9)$$

and express Ψ as :

$$\Psi\left(x_j,t\right)=R\left(x_j,t\right)\exp\left[\frac{i}{\hbar}S\left(x_j,t\right)\right] \qquad (10)$$

Inserting Eq. 10 in Eq. 9, we readily obtain:

$$\frac{\partial R}{\partial t}+\frac{1}{m}\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j}+\frac{R}{2m}\frac{\partial^2 S}{\partial x_j^2}=0 \qquad (11)$$

which can be rewritten as :

$$\frac{\partial R}{\partial t}=-\frac{1}{2m}\left(R\frac{\partial^2 S}{\partial x_j^2}+2\frac{\partial R}{\partial x_j}\frac{\partial S}{\partial x_j}\right) \qquad (12)$$

and :

$$-\frac{\partial S}{\partial t}=\frac{1}{2m}\left(\frac{\partial S}{\partial x_j}\right)^2+V-\frac{\hbar^2}{2m}\frac{1}{R}\frac{\partial^2 R}{\partial x_j^2} \qquad (13)$$

We now introduce the density of probability of presence, according to :

$$P(x) = |\Psi|^2 = R^2 \qquad (14)$$

Eqs. 12 and 13 can then be rewritten as:

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x_j}\left(\frac{P}{m}\frac{\partial S}{\partial x_j}\right) = 0 \qquad (15)$$

$$-\frac{\partial S}{\partial t} = \frac{1}{2m}\left(\frac{\partial S}{\partial x_j}\right)^2 + V - \frac{\hbar^2}{4m}\left[\frac{1}{P}\frac{\partial^2 P}{\partial x_j^2} - \frac{1}{2P^2}\left(\frac{\partial P}{\partial x_j}\right)^2\right] \qquad (16)$$

We may take the classical limit $\hbar \to 0$ of Eq. 16, yielding:

$$-\frac{\partial S}{\partial t} = \frac{1}{2m}\left(\frac{\partial S}{\partial x_j}\right)^2 + V \qquad (17)$$

which is exactly Eq. 1 of the Hamilton-Jacobi's formulation of classical mechanics.

Then, Eq. 3, which defines the momentum of classical particles in classical mechanics, is extended to hidden trajectories in quantum mechanics:

$$v_j = \frac{1}{m}\frac{\partial S}{\partial x_j} \qquad (18)$$

This is the guidance formula of pilot wave theory. Below the quantum level associated with the wave function Ψ, there is a deterministic sub-level of classical-like trajectories. The velocity of the particles, as seen from Eqs. 10 and 18 is orthogonal to the phase of Ψ which, therefore, plays the role of a pilot wave.

Now, from Eq. 15, we can derive a continuity equation reading as:

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x_j}\left(P v_j\right) = 0 \qquad (19)$$

This equation allows one to regard P as a density of probability of presence for particles in an ensemble of trajectories. And, according to Eq. 18, these trajectories are orthogonal to is o-value surfaces of S.

Going back to Eq. 16, we see that the motion of the particle can be viewed as depending on two potentials: the classical potential V and another extra-potential U, called the quantum potential, reading as :

$$U = -\frac{\hbar^2}{4m}\left[\frac{1}{P}\frac{\partial^2 P}{\partial x_j^2} - \frac{1}{2P^2}\left(\frac{\partial P}{\partial x_j}\right)^2\right] = -\frac{\hbar^2}{2m}\frac{1}{R}\frac{\partial^2 R}{\partial x_j^2} \qquad (20)$$

The equation of motion of the particle then takes the form of a generalized Newton's law according to:

$$m\frac{d^2x_j}{dt^2} = -\frac{\partial}{\partial x_j}\left(V - \frac{\hbar^2}{2mR}\frac{\partial^2 R}{\partial x_j^2}\right) \qquad (21)$$

Now, a most important fact is that the pilot wave has been made empirically equivalent to quantum mechanics. Following Bohm (see also Holland [18] for a similar discussion), *all the results of the usual interpretation are obtained from our interpretation if we make the following three special assumptions which are mutually consistent:*

- *(1) That the Ψ -field satisfies Schrödinger's equation*
- (2) That the particle momentum is restricted to $p_j = \partial S / \partial x_j$.
- *(3) That we do not predict or control the precise location ofthe particle, but have, in practice, a statistical ensemble with probability density $P = |\Psi|^2$. The use of statistics is, however, not inherent in the conceptual structure, but merely consequence of our ignorance of the precise initial conditions of the particle.*

Therefore, we again have, in this example, two empirically equivalent theories with conflicting ontologism :the orthodox quantum mechanics exhibiting an intrinsic indeterminacy, and the heretical pilot wave relying on the deterministic motion of hidden trajectories. In the pilot wave framework, indeterminacy is no more of an intrinsic nature. It is simply the consequence of our lack of knowledge concerning hidden trajectories. Probabilities are no more intrinsic, but now receive a classical interpretation, like when tossing a coin, where our inability to predict the outcome of the toss merely reflects our ignorance of initial conditions (andour inability to carry out computations which are too much complicated and CPU-demanding).

As still another comment, a colleague of mine (the same as the one before) denies the fact that this second case-study is a decent case of ontological under-determination because the usual quantum mechanics does not describe the world but only the observations we can make of the world. From this point of view, the usual quantum mechanics would not count as a theory or, at least, it is not an ordinary theory insofar as it does not describe the furniture of the world, i.e. it does not present any ontology. This is certainly the point of view which was shared by Bohm himself too as testified by the fact that he considered his approach as an ontological interpretation of quantum mechanics [28], explicitly pretending that the usual quantum mechanics is not ontological. This is however not appoint of view which would be shared by the many defenders of quantum mechanics, at least by those who would accept the existence of a, fuzzy or not, ontological quantum reality. For these defenders, there is at least one ontological entity of quantum mechanics, namely the wave-function Ψ ,or more generally the state vector $|\Psi\rangle$, which, although unobservable, provides in a complete way all the information required to compute everything which can be observed.

Undesirability

Now that we have in mind two examples of ontological under-determination (Newton versus Hamilton-Jacobi, and Copenhagen versus pilot wave), it is worthwhile to revisit our understanding of the issue. We are going to do it under another cover, the one of undesirability, because the diagnosis of an under-determination by experiments (whether ontological or not) is the same as a diagnosis of undesirability. In this section, we shall focus on the undecided ability between usual quantum mechanics and pilot wave. It has been argued that the pilot wave is simply quantum mechanics recast din another language and, if this were true, the issue of undecided ability could be of little significance. We could say: just use the language you have learnt, excepted if you are inquisitive enough to feel the desire to learn another language. But the fact is that, when you learn another language (for instance English when you are French, or more convincingly Japanese or Chinese when you are French or English), you really enter into another world, facing a quite different way of thinking.

And, if this is true for natural languages, it is still true, or even more, when we compare quantum mechanics and pilot wave. These two theories offer different visions and interpretations of the world. The undesirability between both theories is therefore also undesirability between two different visions of the world. This undesirability may also be expressed in the Popper's framework of objective knowledge because, let us recall it, both theories may be simultaneously corroborated or falsified.

This is a severe blow against realism, in some sense, that is to say against the very idea that science could pretend to really tell us something on the world around us. This might drive us toward cultural relativism, a philosophical position that is, nowadays, praised by many philosophers. But we have to be careful and avoid to draw hastily erroneous conclusions. What our case-study does imply is not an absolute relativism. It precisely tells us the following: in some cases, we may be facing some undividable statements because they pertain to undividable and even contradictory theories, which have however to be simultaneously accepted. It does not tell us that undesirability is the ultimate fate in *all cases,* nor forever, or *in all details*. For instance, both quantum mechanics and pilot wave agree on the non-locality of microscopic phenomena (in agreement with experiments). In balanced terms, the consequence of any undecidability between two theories concerning realism is that we must abandon the naive scientist project to know the world in all aspects, although some of the aspects of the veiled reality could be unveiled [40].

The most significant undecidable issue concerning quantum mechanics and pilot wave is the one of determinism, actually an issue which boosted the search for hidden variables. Indeed, if we cannot decide between quantum mechanics and pilot wave, we cannot decide between the intrinsic indeterminacy of quantum mechanics and the determinism postulated, and afterward constructed, by causal theories. Both Einstein (an opponent to quantum mechanics) and Born (a defender of quantum mechanics) even agreed on the undesirability between determinism and indeterminism. In a letter to Einstein, Born wrote: *You are absolutely right that an assertion about the possible future acceptance or rejection of determinism cannot be logically justified. For there can always be an interpretation which lies one layer deeper than the one we know (as your example of the kinetic theory as against the macroscopic theory shows)*[41].

Bohm used a similar argument and explicitly extended it to defend the undesirability under question. For him, starting from a deterministic (in deterministic) level of description, we can always imagine and construct a sub-level which would be in deterministic (deterministic), and this *ad infinitum*. For instance, determinism at a certain upper level may be the result of in deterministic, or possibly stochastic, processes at the next lower level, e.g. macroscopic determinism resulting from averages over quantum indeterminist processes or atomic classical stochastic processes, at a microscopic level. And indeterminism at a certain upper level may be the result of deterministic processes at the next lower level, e.g. quantum indeterminacy sustained by deterministic hidden variables. In the words of Bohm himself [27], *... one sees that the possibility of treating causal laws as statistical approximation to laws of chance is balanced by a corresponding possibility of treating laws of probability as statistical approximations to the effects of causal laws... The assumption that any particular kind of fluctuations are arbitrary and lawless relative to all possible contexts, like the similar assumption that there exists an absolute and final determinate law, is therefore evidently not capable of being based on any experimental or theoretical developments arising out of specific scientific problems, but it is instead a purely philosophical assumption.*

Interestingly enough, Bohm and Hiley[28] took advantage of the issue of undecid ability to advocate a peaceful relationship between Bohm's interpretation and quantum mechanics. For them, and also for us at the present time, *there does not seem to be any valid reason ... to decide finally what would be the accepted interpretation*, and they asked :*is there a valid reason why we need to make such a decision at all?* So, they went on, *would it not be better to keep all options open and to consider the meaning of the interpretations on its own merits, aswell as in comparison with others? This implies that there should be a kind of dialogue between different interpretations rather than a struggle to establish the primacy of any of them*. If we observe that these quotations are contained in the last book of Bohm (with Hiley), published one year after his death, we can view the above statements as forming a kind of final testamentary auto-appraisal.

Beyond undesirability, applicative arguments

But here is now some kind of magic trick: we can actually, at least in some cases, decide between undecidable physical theories. When this is possible, this is achieved by using what is called applicative arguments. We are now going to discuss the concept of applicative arguments by referring ourselves to Harré.

According to him [5], *a theory is plausible if it is both empirically adequate and framed within the constraints of the current communally approved ontology*. Discussing more extensively the concept of plausibility (and the one of implausibility), Harré exposed five conditions for plausibility. Rather than following Harré, I shall simplify my exposition by referring to the rewording of Van Fraassen[9]. Following Van Fraassen, a theory is definitely acceptable for Harré if it satisfies two conditions (1) it must be in agreement with empirical facts (2) it must be plausible. In the mind of Harré, plausibility means that the theory must imply mechanisms and entities pertaining to the unique hierarchy of ontological types underlying the history of the scientific enterprise. I am accepting these two criteria as they are expounded above, but I shall discuss the plausibility demand in a way different from the one of Harré and VanFraassen. Actually, the explanation of the word "plausibility" as given by Harréand Van Fraassen may be found unsatisfactory. It looks to me too much conservative insofar as it refers to an approved ontology or to a hierarchy of ontological types related to the past of the history of sciences. But, in the actual enterprise of sciences, new entities may have to be built and old entities may have to be destroyed. Such new entities do not pertain to ontological types already used previously in the history of sciences. Therefore, old ontologies may have to be destroyed and new ontologies may have to be built.

The first demand of Harré, in Harré's list (reworded by Van Fraassen), concerning the agreement with empirical facts, does not require extensive discussions. However, we can decompose the second demand of Harré into sub-demands. The first sub-demand (denote it as 2a) requires theological consistency of an acceptable theory. It is indeed plausible (even more than plausible!) that a satisfactory theory must be logically consistent. With the demands we have retained up to now, namely demand 1 for agreement with experimental facts and demand 2a for logical consistency, weave enough to state a possible formulation of Quine's under-determination thesis: two theories which are logically consistent, and observationally equivalent, may be contradictory. We may now introduce a second sub-demand in the plausibility criterion of Harré, let us call it demand 2b. The demand 2b states that, besides being logically consistent and making predictions agreeing with experiments, a satisfactory theory must also satisfy other demands (to be discussed below). These other demands, let us call them applicative arguments. The word "applicative" means that these arguments enlarge our possibilities of choosing between several theories, the enlargement being made with respect to the under-determination by experiments. In other words, applicative arguments allow us to decide between undecidables. Therefore, we may now build a new list of criteria as follows.A theory is satisfactory if (1) it agrees with experimental facts (2) it is logically consistent (3) it satisfies applicative arguments.

It is interesting to remark that Einstein had something to tell us concerning Quine's under-determination (without referring to it) and ampliative arguments (without using this terminology). Indeed, in an address he delivered in 1918 before the Physical Society of Berlin on the occasion of Planck sixtieth birthday, he expressed himself as follows [11]: *there is no logical paths in these laws; only intuition, resting on sympathetic understanding of experience, can reach them. In this methodological uncertainty, one might suppose that there were any number of possible systems of theoretical physics all equally well justified; and this opinion is no doubt correct, theoretically. But the development of physics has shown that at any given moment, out of all conceivable constructions, a single one has always proved itself decidedly superior to all the rest. Nobody who has really gone deeply into the matter will deny that in practice the world of phenomena uniquely determines the theoretical system, in spite of the fact that there is no logical bridge between phenomena and their theoretical principles.*

Naughty applicative arguments

The question to know what are acceptable applicative arguments is a difficult one which might better be postponed to future epistemological researches, but the question to know what are unacceptable applicative arguments may be more easily answered, at least partially. For this, I shall discuss some reasons (ampliative arguments), all of them to be viewed as naughty, which have been used to discriminate between Bohm's pilot wave and usual quantum mechanics, leading to the conclusion, if they were accepted, that Bohm's approach must be rejected. To this purpose, I shall refer to Bitbol[42] who provided five reasons, expressed by various authors, to motivate the rejection of Bohm's pilot wave, and I shall discuss these reasons, although in a brief but, I believe, sufficient way.

Discussions that are more comprehensive might be welcome but would extend beyond the scope of the present paper. The first reason is the positivistic accusation of metaphysics, anchored in a long lasting philosophical tradition devoted to the dissolution of metaphysics. For the defenders of the orthodox Copenhagen interpretation, at least for some of them, the arrogant accusation of metaphysics helped to push away hidden variables toward the darkness of a medieval heretical way of thinking.

This was most often motivated by the fact that the introduction of hidden variables did not provide any new prediction, but introduced an unobservable "superstructure". Against this accusation, Bohm's defense maybe received [43]: *Past experience in a wide variety of fields would, however, suggest the falsity of such a conclusion; for it has very often turned out to be very fruitful indeed to postulate the existence of things before anyone knew, even in principle, how to go about trying to verify their existence.* Let us add to this that the wave function Ψ of quantum mechanics is unobservable and, therefore, in the eyes of some positivists, should be granted as metaphysical as well.

A second reason concerns the undesirability between pilot wave and quantum mechanics. Starting from the fact that the usual quantum mechanics is dominant (Bolshevik), we should reject the marginal (Menshevik) pilot wave since there is no reason, due to undesirability, to choose it. This is indeed a naughty reason: If you cannot decide, you cannot reject. The third reason is that there exist other less adventurous means to preserve a realist conception of quantum mechanics, such as the one of the veiled reality. This reason is naughty too because it is evidently too much subjective; also, being adventurous is not necessarily an inefficient attitude. The fourth reason is that the associated research program exhibits regressive character. This is another naughty reason. The question is not necessarily to know whether the pilot wave theory is regressive when compared with quantum mechanics taken as a standard, but it could be to know whether quantum mechanics is not unduly hazardous with respect to classical concepts. The fifth reason is that the early Bohm's theory, the one dated 1952 [26], [39], even did not succeed to keep on with its own spirit which, initially, would have been of an atomist nature. In agreement with this spirit, it starts with a view of the world made out from particles which are individual entities on their own, equipped with properties having a classical flavor, but has afterward to accept highly on classical features (such as non-locality and conceptuality). This is also a naughty reason. Indeed, it happens very often that a researcher starts a work with some preconceptions and prejudices indicating a certain direction and that, eventually, the landscape reached does not look like the landscape expected. This is a matter of fact and a matter of life, not a basis for rejection.

To Bitbol's list, we may add two still naughty applicative arguments, increasing our total to seven naughty reasons. The first one, advocated by the defenders of the usual quantum mechanics, invoking the famous Occam razor principle, is a lack of simplicity of the pilot wave with respect to the usual quantum mechanics. Bohm defended himself [28] by stating that, although his interpretation has the additional assumption of particles, this is balanced by the fact that it does not require the usual assumption of probability. So, he said, at least on the basis of a formal count of the number of assumptions, it cannot be concluded that either interpretation (pilot wave or Copenhagen) is favored over by the principle of Occam's razor. More important however, the demand of simplicity cannot be a first principle of physics, and there is no reason to believe that it would be correct to apply it whatever the circumstances. Indeed, there exists a famous counter-example, namely the demand for circular motions of the planets (because the circle is the simplest curve producing spatially finite motion) which drove astronomy toward the epicycles of Ptolemy, before it became possible to escape from this trap. Moreover, finally, for the seventh and last item, it has been argued that Bohm's theory is incomplete. This is true, but our current physics is incomplete too.

I am now going to propose another applicative argument which, I hope, is not naughty. In any case, for the sake of charity, I can provide loopholes. Nevertheless, the argument is certainly convincing enough, at least for many individuals, as an example, but also as our tool to proceed further, beyond what was allowed to us by the mere use of Quine's under-determination. This example relies on a non-singularity principle which, if accepted, implies the falseness of Newton's formulation of classical mechanics (more generally, the inadmissibility of classical mechanics), the rational necessity of wave mechanics, and the inadequacy of the pilot wave theory.

Non-singularity principle and consequences

Non-singularity principle

I am now going to summarize the contents of a recently published paper [44] in which, after a discussion of the concept of infinity in mathematics and in physics, a non-singularity principle is stated, telling us that local infinity in physics is not admissible. The reader is kindly requested to refer to this paper for details.

A first example of application concerns the optical rainbow [45], [46], [47]. The simplest way to understand the basic features of the optical rainbow is to start with geometrical optics, more precisely with ray tracing, an approach usually granted to Descartes, although there are precursors. The existence of the rainbow then comes from the fact that the deviation of the once internally reflected ray passes through a minimum when the angle of incidence is varied (this is called stationary ray). The concentration of rays, near the stationary ray, generates a singularity which is a real caustic, separating a bright side from a dark side. More generally, singularities are predicted by geometrical optics at focal points, lines, and caustics. The word "caustics" can be viewed as a generic word to refer to any kind of singularity produced by ray families filling regions of space. Invoking the non-singularity principle, we may then conclude that geometrical optics, in utmost rig our, is inadmissible and that it actually must be an approximation to a more general theory which has to be a wave theory (because waves remove singularities).Indeed, such is the fact : Light waves are described by the vectorial Maxwell's equations, and the exact theory of the rainbow is provided by the Lorenz-Mie theory [48], [49], [50] which describes the interaction between a sphere and an illuminating electromagnetic plane wave. We shall return to Lorenz and Mie for our third case-study.

For a second example, we now consider classical mechanics. A sub-topic of classical mechanics is classical scattering, e.g. [51], [52] which, in contrast with electromagnetic scattering, is scalar scattering instead of being a vectorial scattering. Now, it happens that, similarly as for the optical rainbow in geometrical optics, there exists a singular mechanical rainbow in classical scattering [52]. The non-singularity principle then implies that classical mechanics has to be rejected, and viewed as an approximation to a more general theory removing the singularity of the mechanical rainbow, namely a wave mechanics.

The non-singularity principle is now going to be used as an applicative argument. Nevertheless, let us remark that there are several possible loopholes to reject the non-singularity principle, and its consequences [44]. For instance, we may refer to Quine's epistemology [7], [53], [54] according to which any statement is in principle revisable, even any logical statement. This is in agreement with the fact that an applicative argument is not necessarily meant to force the adhesion of every one. However, from now on, in the sequel, shall hold fast on the non-singularity principle as a quite decent applicative argument to proceed further. In particular, an immediate consequence of the non-singularity applicative argument is that Newtonian trajectories of matter points do not exist.

Newtonian trajectories of matter points do not exist.

We are all used to the concept of trajectory, so easy to extract from classical mechanics, in deep agreement with our intuition, and with our sense data: trajectories of cars, of balls, or even of planets. There is no apparent difficulty to consider trajectories of matter points. They at least constitute efficient models for the behavior of so many objects around us. We therefore feel very comfortable with the concept of trajectories in the Newton's formulation of classical mechanics and, as a consequence, we may feel very uncomfortable with the Hamilton-Jacobi's formulation in which trajectories are dressed by a field. What could physically be this field S, which extends in the whole space around the object under motion that no one has ever seen and which is effectively unobservable? At the best, it is a convenient intermediary for computations and, at worst; it is some kind of metaphysical artificial excrescence. At least, this is what we could instinctively believe.

But, actually, although this way of thinking seems very reasonable, it may be dramatically misleading. Let us effectively consider the non-singularity principle and its application to classical scattering. It leads us to the rejection of classical mechanics, as being inadmissible, on the basis that, in some cases, a classical problem may produce singularities, in particular rainbow singularity. If we hold fast on this conclusion and use it as an applicative argument, we reach an immediate consequence: Newtonian trajectories of matter points do not exist (and cannot exist).This statement is actually in deep agreement with sense data. Effectively, no one ever observed the trajectory of a matter point. Simply try to imagine, at the most fundamental level of understanding, what could be the trajectory of a matter point, that is to say the trajectory of an object of mass m, confined to a vanishingly small volume. Such an object would have an infinite density, and Newtonian mechanics would have to be merged with the most extraordinary predictions of general relativity. There would not be any more any domain of validity for classical mechanics.

A special kind of trajectory is the motionless trajectory of a particle at rest. Then, we have the particle (the matter point) standing still (in some frame of reference). From this, we can see that the concept of a matter point, independently of the concept of trajectory, cannot be accepted. Beside what we have already said, including the idea of a collapse of a mass m to a geometrical point to generate a matter point, just think, for instance, of the unbelievable behavior of the electrostatic potential of a point charge varying as $1/r$, and therefore diverging when the point charge is indefinitely approached (producing an actual infinity). Obviously, from a pure logical point of view, we might as well start to refute first the existence of matter points invoking the non-singularity principle, and there after the existence of trajectories of matter points, since something which does not exist cannot have any trajectory.

Furthermore, let us remark that, when teaching students, it is often heavily pointed out that trajectories do not exist in quantum mechanics. This may be viewed as a consequence of Heisenberg's uncertainty relations, forbidding the simultaneous measurements of locations and moment a, these quantities which are so vital for the very possibility of defining a trajectory. In addition, the students are supposed to be surprised by such a rupture. But there should not be any surprise there. If trajectories of matter points already do not exist in classical mechanics, why should they exist in quantum mechanics?

Nevertheless, what about trajectories of cars, balls ... and planets? The answer is that there should not be any deep and definitive conflict between the non-existence of trajectories of matter points, and the observe ability of trajectories of extended solids. Cars, balls … planets are not matter points that we cannot observe, but macroscopic objects that we do observe. Their existence, their properties, the fact that we can observe them, and by which processes we observe them, must emerge as the consequences of a quantum theory, whatever its ultimate formulation will be. In other words, classical Newtonian trajectories of extended objects have to be accepted, but such extended objects, rather than being considered as a collection of matter points, in the Newton's style, must emerge from a more fundamental description of nature, e.g. from the underlying quantum mechanical level. Hence, the rejection of matter points, and of their trajectories, does not offend our everyday intuition.

Deciding between undesirables

The fact that Newtonian trajectories of matter points do not exist is the ultimate applicative argument to discriminate between Newton's and Hamilton-Jacobi's formulations of classical mechanics. In contrast with our naive expectation in which the field S was viewed as a simple intermediary tool for computations, without any physical significance, it is then Estonian formulation of classical mechanics which is to be rejected, and the Hamilton-Jacobi's formulation which is to be given a due privilege, let us say which is "closer to truth". We are then left with a dressing field S without any trajectory to be dressed. To understand such a weird situation, it is sufficient to remark that, not only Newtonian trajectories of matter points do not exist but, also, it is classical mechanics as a whole which collapsed. Hence, Hamilton-Jacobi's formulation of classical mechanics is to be rejected too. But, nevertheless, the field S still remains physically meaningful. It actually appears to be related to the phase of Ψ in quantum mechanics. Indeed [16], we may write Ψ under the form:

$$\Psi = \exp\left(\frac{iS}{\hbar}\right) \quad (22)$$

in which, in the classical limit, the quantum field S of Eq. 22 identifies with the classical field S of Hamilton-Jacobi's formulation. The physical meaning of the classical S of the inadmissible classical mechanics is therefore that it constitutes a formal anticipation of the Ψ of quantum mechanics, more precisely of its phase. The constant $\hbar$, which has the dimension of an action, has the virtue of changing S, which has the dimension of an action too, to a dimensionless phase $S/\hbar$.

Concerning the second case-study (pilot wave versus usual quantum mechanics), the relevance of Quine's under-determination for a discussion of Bohm's pilot wave has been noticed by Cushing [55], [34]. His point of view is that [55]*one formalism, with two different interpretations, counts as two different theories* and that [34]*the physical interpretation refers to what the theory tells us about the underlying structure of ...phenomena, i.e. the corresponding story about the furniture of the world*. This furniture of the world is that he called an ontology (the point of view also adopted in this paper). Therefore, quantum mechanics and Bohm's pilot wave, although experimentally equivalent (implying that both of them *should* be accepted by positivists), provided two different ontologies. Bohmhim self was aware of the issue at least implicitly [27].

However, we may implicatively discriminate between the pilot wave and the usual quantum mechanics. For this, we just need to remark that the same applicative arguments (non-singularity principle, inexistence of Newtonian trajectories of matter points) which have been used to discriminate between Newton's and Hamilton-Jacobi's formulations of classical mechanics may be used to discriminate between Bohm's pilot wave and quantum mechanics. If objective, deterministic, Newtonian trajectories of matter points (and matter points themselves) do not exist, it is a non sense to introduce the min quantum mechanics as done by Bohm. The attempt to propel the classical concepts of matter points and matter point trajectories in the quantum domain leads definitively to a failure. Trajectories of matter points do not exist in classical mechanics; hence, they should not be reintroduced in quantum mechanics, even if they are hidden. This statement applies to allcausal theories (pilot wave of Louis de Broglie and Bohm, and double solution of Louis de Broglie).

The story of the pilot wave does not stop with the original 1952-version of Bohm. Many other developments have been further elaborated by Bohm himself, alone or with collaborators, or by other independent researchers, inspired by causal theories. A story of these further developments, and of the many criticisms they receive, is outside of the scope of this paper. What is however important with an applicative approach, as used here, is that it is an upstream argument which dries up the flow at the source. It is indeed much more comfortable to possess an upstream final objection than having permanently to fight downstream with non convincing objections, facing growing flow flooding all objections and counter-objections. Causal theories were like the Leonean Hydra with cut heads growing again. An upstream argument cuts all the heads in one stroke. And this upstream argument just tells us that everything was basically flawed from the very beginning.

The rejection of inadmissible theories is not contradictory with the fact that they can make very decent models, much useful in practice. And, after all, our best theories are very far from being perfect and completed: from this point of view, they are all models. In particular, the fact those singularities in the behavior of Newtonian trajectories are rare, occurring only occasionally, demonstrate that the concept of trajectory of matter points still remains useful for many practical purposes, in the same way that optical ray computing and tracing will forever remain invaluable tools. But, if we want to dig deep into the mysteries of the world, it indefinitely of good advice, and even compulsory, to abandon inadmissible theories, when possible. The idea, expressed by Bohm in its auto-appraisals, and by other authors, that the pilot wave could provide complementary insights to a better, more thorough, and deeper, understanding of quantum mechanics, is then erroneous and even dangerous, for there could not be complementary insights in erroneous ideas, just only an opportunity to spoil the clarity of the mind.

Any theoretician in quantum field theory, for whom the description of reality in terms of particles being permanent entities with fixed numbers is more than naive, would agree with this statement. It is nevertheless very important to remark that the rejection of Bohm's mechanics is not, from a logical point of view, to be taken as an approbation of the usual quantum mechanics (the one of the text-books).Indeed, many scientists are still dissatisfied with quantum mechanics, as it stands now, and we cannot exclude the possibility of a new forthcoming breaking approach.

Third case-study: from electromagnetism

We have previously examined two case-studies which are strongly correlated. One pertains to classical mechanics, the other to quantum mechanics, but both of them were anchored on the existence of the Hamilton-Jacobi's formulation of classical mechanics, and undesirables were made decidable by relying on similar applicative arguments. We are now going to briefly discuss third case-study, related to another physical framework. We are going to solve a question that I asked myself about thirty years ago, a long time before I heard from Quine's under-determination and from applicative arguments.

This case-study concerns classical electromagnetism, and particularly light scattering theory. In this field of research, the most famous theory is likely to be a theory published by Gustav Mie in 1908 [50]. This theory describes the quasi-elastic interaction (no change of frequency except that one due to the Doppler Effect, and the other singular one from finite frequency to a "null" frequency due to absorption) between an illuminating electromagnetic plane wave and a homogeneous sphere defined by its (arbitrary) size and its (arbitrary) complex index of refraction. It allows one to calculate scattered fields outside of the sphere, internal fields, phase relations, various cross-sections (for scattering, absorption, and extinction), and radiation pressure forces and torques. The theory is built by using Maxwell's electromagnetism. The significance of this theory may be appraised by the fact that it is still regularly and even increasingly cited. The year 2008 was the year of the hundredth anniversary of Mie's paper which has then be commemorated in several conferences (i) GAeF conference on "Light scattering : Mie and More", 3rd and 4th July 2008, in Karlsruhe, Germany[56] (ii) 11th conference on electromagnetic and light scattering, 7th-12th September 2008, in Hatfield, UK [57] (iii)"Mie theory 1908-2008 : present developments and interdisciplinary aspects of light scattering", 15th-17th September 2008, University of Halle-Wittenberg, Germany and (iv) International Radiation Symposium IRS2008, 3rd-8th August, Foz do Iguaçu, Brazil [58].Furthermore, it has been successfully generalized to the case when the illuminating beam is a laser beam, with many applications in various fields [59].

But, actually, twenty years before Mie's paper, a similar theory had been produced by Lorenz (Lorenz, not Lorentz) [48], [49]. Hence, rather than speaking of Mie's theory, I always preferred to use the denomination of Lorenz-Mie theory. Several historical papers are available concerning the formulation of Lorenz, and its relationship with the one of Mie [60], [61], [62], [63].According to these references, Mie's and Lorenz' theories lead to the same experimental predictions, that is to say, they are experimentally equivalent. The work of Lorenz has been overlooked, certainly in part because it has been written in Danish and, also, due to the fact that an important memoir has been lost. But, more relevant to our subject, it did not rely on Maxwell's equations. Indeed, it did rely on a theory of a ether.

In the previous case-studies, we have been able to expound the mathematics associated with the physics. From a technical point of view, this is not possible in the present case-study because the mathematics required are too involved, e.g. [59].Therefore, regarding the present third case-study, the reader may feel frustrated insofar as he cannot view by himself all the details involved, and has to rely on experts quoted in the references. Unfortunately, when dealing with difficult matters in science, and in philosophy of science, relying on experts may be necessary. This should not prevent us to append an item to a list of future research problems, namely revisiting both Lorenz and Mie theories, and their connection as well. But nothing more about this issue can reasonably be expounded in this paper.

Nevertheless, more importantly, from the philosophical point of view of interest to us in the present paper, we then may state that, therefore, the two theories are indeed empirically equivalent *but* they do exhibit two different visions of the world, one with anther, the other one without any anther. In the mechanical approach used by Lorenz, the anther was viewed as the support of what are now named "light and other electromagnetic radiation" [45], to echo the title of a famous book by Kerkerdevoted to light scattering.

This support has to be considered as an ontological entity, in the same way that water, the support of water waves, can be considered as an ontological quantity : water, whatever it is *in fine*, does exist; it does pertain to the furniture of the world. In deep contrast, Maxwell's electromagnetism does not invoke any a ether: electromagnetic waves are viewed as waves without any support. Hence, we are here once more facing a case of ontological under-determination.

We may again rely on an applicative argument to discriminate between Lorenz and Mie. This argument simply invokes the well-founded rejection of a ether by Einsteinium relativity. Hence, from this point of view, Mie was "closer to truth" than Lorenz. Considering the effort accomplished by Lorenz, he nevertheless certainly deserved to have his name associated to what I always preferred to call Lorenz-Mie theory. However, the two theories, although empirically equivalent, are ontologically different, and may better have to be viewed as two genuine different theories.

I would like to end this section by commenting an interesting complementary discussion by Bell [64] comparing the approaches of Lorentz (Lorentz, not Lorenz) and of Einstein to the issue of Lorentz invariance. In short, according to Bell, there is a difference of philosophy (and difference of style that I am not going to consider). "The difference of philosophy is this. Since it is experimentally impossible to say which of two uniformly moving systems is really at rest, Einstein declares the notions 'really resting' and 'really moving' as meaningless. For him only the relative motion of two or more uniformly moving objects is real. Lorentz, on the other hand, preferred the view that there is indeed a state of *real* rest, defined by the 'a ether', even though the laws of physics conspire to prevent us identifying it experimentally". Bell then added that the facts of physics do not oblige us to accept one philosophy rather than the other one or, using the terminology introduced in this paper: the facts of physics do not oblige us to accept one ontology rather than the other one. In this case, deciding between undesirables is the consequence of an applicative argument making us rejecting the notion of a ether as useless, if not meaningless. There is however still some room forthe ones who would prefer to preserve the concept of a ether : as usual, an ampliative argument is not necessarily to be accepted by everyone.

Conclusion

Quine's under-determination thesis, loosely speaking, states that theories are under-determined by experiments. I explicitly introduced an ontological version in which we may have several empirically equivalent theories, with however different ontologism. We have examined three exemplifying case-studies. Two of them (Newton’s formulation versus Hamilton-Jacobi's formulation of classical mechanics, causal theories versus the orthodox interpretation in quantum mechanics) are closely related to the debates on the foundations of quantum mechanics. The third case-study (Mie's theory versus Lorenz' theory) is borrowed from electromagnetic theory, more specifically from light scattering theory. Each case-study exhibits a couple of theories which lead to identical experimental predictions but are contradictory insofar as they do provide conflicting visions of the world. The existence of conflicting empirically equivalent theories implies, in principle, strong limitations to any realistic interpretation of science. However, in each case-study discussed in this paper, we have been able to invoke applicative arguments allowing onto decide between undesirables.

Finally, it should be clear that the points of view taken in this paper are the one of a physicist and, surely, the points of view of physicists may be useful to philosophers. Therefore, a philosopher who would be an expert to the understanding of Quine analytical philosophy (thesis of under determination of theories by experiments, inscrutability (or indeterminacy) of reference, and indeterminacy of radical translation) could very likely take advantage of the examples presented in this paper as a support for further epistemological researches and discussions.

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