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Reductio ad Absurdum, Modulation, and Logical Forms

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Abstract

There is a logical valid inference that people usually consider as incorrect and in which the Reduction ad Absurdum rule is involved. That inference consists of a premise and a conditional conclusion in which the antecedent is the denial of the premise. The mental models theory can explain this fact and why, in certain cases, individuals do accept the mentioned inference. However, it seems that the syntactic frameworks can only explain why people often reject the inference, but not why they admit it in some circumstances. In this paper, I try to show that individuals really never consider the indicated inference to be valid, that they only accept inferences similar to it, but not identical to it, and that it can hence be said that these problems do not prove that the syntactic approaches do not hold.

Keywords: formal rules; mental models; Reductio ad Absurdum; semantics; syntax

1. Introduction

At present an interesting theory is proving to be very successful in cognitive science. That theory is the mental models theory (e.g., Byrne & Johnson-Laird, 2009; Johnson-Laird, 1983, 2001, 2006, 2010, 2012; Johnson-Laird & Byrne, 2002; Johnson-Laird, Byrne, & Girotto, 2009; Oakhill & Garnham, 1996; Orenes & Johnson-Laird, 2012) and its approach is basically semantic. According to it, reasoning does not follow syntactic rules. Human inferential activity is based on analyses of the semantic possibilities of propositions. By means of such analyses, individuals can identify the models that are compatible both with the premises and with the conclusions, and those models are the factors that lead deductions, inferences, and derivations. The success of the mental models theory is rooted in the fact that it has demonstrated to have a clear ability to explain and predict most of cognitive phenomena related to reasoning. For this reason, it is undeniable that this theory can be correct, and the goal of this paper is not to prove otherwise. My aim is very more limited. I will only focus on a relevant experiment carried out by two proponents of this theory (Orenes & Johnson-Laird, 2012) in order to show that the mental models theory can explain and predict results that the syntactic theories (fundamentally based on formal rules) cannot. The issue reviewed in the mentioned experiment refers to the 'paradoxical inferences'. Really, they are not paradoxical inferences, since they are logically valid, but that expression is used because they are inferences that, despite, as said, they are logically correct, people tend to reject. Orenes and Johnson-Laird (2012) analyze three inferences of this kind, but the interesting one here is the following:

$$\neg\alpha \vdash \alpha \to \beta$$

'$\neg$' is the logical denial, '$\vdash$' indicates that the left formula deduces or proves the right formula, and '$\to$' stands for the logical conditional. However, this inference is problematic not only because, as mentioned, people do not often accept it, but also because it appears that individuals clearly admit it in some cases in which α and β have certain thematic content. In the mental models theory view, the explanation of these facts can only be semantic, and Orenes and Johnson-Laird (2012) give an account of that kind based on that theory.

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But the most important point for this paper is that, although Orenes and Johnson-Laird (2012) seem to acknowledge that the syntactic theories can also explain why people generally reject these inferences, in the same way they appear to state that the explanation of why the inferences are accepted in certain cases cannot be based on a syntactic account, since it is a semantic fact. As said, I will not try to question or undermine the mental models theory. I only wish to prove, in the next pages, that Orenes and Johnson-Laird´s (2012) experiment do not demonstrate that there is a phenomenon that can be explained by the mental models theory and not following a formal view. My main thesis is that the versions of this inference that are often accepted do not truly have the same structure or the same logical form as the versions that tend to be rejected. Thus, I will argue that Orenes and Johnson-Laird´s (2012) experiment cannot be considered to be a crucial experiment that proves that the mental models theory is correct and that the syntactic frameworks do not hold, at least with regard to the type of inference indicated above. To do this, firstly, I will comment in more details the characteristics of that inference and the difficulties linked to it. Secondly, I will expose the accounts that can be given both from the mental models theory and from syntactic approaches in order to explain why individuals tend to reject the inference. Thirdly, I will describe Orenes and Johnson-Laird´s (2012) explanation of why the inference is accepted under certain circumstances and indicate which, according to them, those circumstances are. Finally, I will try to show that the versions that usually are admitted are not really inferences of the same kind as those that are generally rejected, and that Orenes and Johnson-Laird (2012) experiment is not the best experiment for proving that human beings think by means of mental models.

2. The Problem

The problem is, as mentioned, that people do not often admit inferences such as this one: "It won´t rain today. Does it follow that if it does rain today then the forecast is right" (Orenes & Johnson-Laird, 2012, p. 374). And this a problem because it is clear that the logical structure of this reasoning task is as follows:

$\neg p$ *Ergo* $p \rightarrow q$

And, as also said, $\neg \alpha \vdash \alpha \rightarrow \beta$ is a valid inference. This last statement can be easily checked considering Gentzen´s propositional calculus. If we assume that '$\wedge$' is the logical conjunction, the derivation would be similar to the following:

(1) $\neg \alpha$ (Premise)
(2) α (Supposition)
(3) $\neg \beta$ (Supposition)
(4) $\alpha \wedge \neg \alpha$ ($\wedge$ Introduction 1, 2)
(5) $\neg\neg \beta$ (Reductio ad Absurdum 3-4)
(6) β ($\neg$ Elimination 5)
(7) $\alpha \rightarrow \beta$ ($\rightarrow$ Introduction 2-6)

Thus, if human reasoning followed rules such as those of Gentzen´s propositional calculus, people would generally accept inferences with this logical structure. Nonetheless, Orenes and Johnson-Laird (2012) think that the explanation of this fact is not complex and show how the mental models theory can give an account of it.

3. The Explanation from the Mental Models Theory

According to the mental models theory, the possibilities to which a conditional proposition refers are, in principle, those corresponding to the true cases of the material interpretation of it. Thus, if it is assumed that '1' stands for truth and '0' stands for falsehood, such possibilities are the cases with 1 in Table 1.

Table 1: Truth Table Corresponding to the Material Interpretation of Conditional

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

In this way, if 'v' refers to the truth-value of the formula that follows between brackets, it can be said that $v(p \to q) = 1$ when $v(p) = 1$ and $v(q) = 1$, when $v(p) = 0$ and $v(q) = 1$, and when $v(p) = 0$ and $v(q) = 0$. This hence indicates us that the structures of the models of conditional are these ones:

$$p\&q$$
$$\neg p\&q$$
$$\neg p\&\neg q$$

Note that the structure $p\&\neg q$ is not possible because it corresponds to the only case in which $v(p \to q) = 0$, i.e., to the case in which $v(p) = 1$ and $v(q) = 0$. Therefore, it can also be said that, for the example quoted in the previous section, the models of its conclusion are the following:

It rains - The forecast is right
It does not rain – The forecast is right
It does not rain – The forecast is not right

An important point of the mental models theory is that only $p\&q$ (that is, it rains – the forecast is right) is an explicit model. Individuals must make endeavors to take the other two models into account. Nevertheless, the most relevant aspect for this paper is that, according to Orenes and Johnson-Laird (2012), precisely this explicit model is not consistent with the premise $\neg p$ (that is, it will not rain), and this is the reason that people do not accept inferences such as $\neg\alpha \vdash \alpha \to \beta$. $\alpha \to \beta$ is related to three possibilities, and the first one of them (which is, in addition, the more explicit possibility) is not coherent with the premise. Obviously, $p\&q$ can only be true if both p and q are true, and the premise indicates that p is false. In this way, from the mental models theory perspective, this fact is strong enough to explain why people tend to reject inferences of this kind. Nonetheless, as mentioned, this same fact can also be explained from a syntactic point of view. I will show it in the next part.

4. The Syntactic Explanations

Really, we can give several syntactic explanations of this problem, not only one. Some of them are even commented by Orenes and Johnson-Laird (2012), who mention that Rips (1994) proposes various accounts in this regard. One of those accounts is particularly interesting and is related to the idea that, when a conclusion is less informative than its premise (or premises), that conclusion tends not to be accepted. Evidently, to state $\neg\alpha$ is more informative than to state $\alpha \to \beta$, and, therefore, as Orenes and Johnson-Laird (2012) acknowledge, a theory based on formal rules can also explain the difficulties linked to this paradoxical inference. Besides, there is other syntactic approach that is not considered by Orenes and Johnson-Laird (2012) and that can give its explanation too. The mental logic theory (e.g., Braine & O´Brien, 1998a; O´Brien, 2009) is that approach. According to this theory, human mind indeed works following formal rules, but those rules are not necessarily the rules of calculi such as Gentzen´s propositional calculus. The mental logic theory is based on empirical and experimental findings that seem to show that individuals do not often use certain logical rules. For example, as argued by Braine, Reiser, and Rumain (1998), because empirical evidence indicates that people do not usually apply the disjunction introduction rule, i.e., the rule $\alpha \vdash \alpha \vee \beta$ ('$\vee$' refers to the logical disjunction), the mental logic theory claims that individuals cannot be expected to use this rule in most cases. In the same way, this theory states that, although a contradiction such as $\alpha \wedge \neg\alpha$, which is needed for the application of the *Reductio ad Absurdum* rule, allows deriving any formula in Gentzen´s propositional calculus, such a contradiction only means that nothing can be drawn or that some premise is false in mental logic (Braine & O´Brien, 1998b).

This last point is important for the problem of this paper. Johnson-Laird (2010) says that contradictions are a difficulty for the syntactic theories, since, in principle, every formula can be deduced from a contradiction. Nevertheless, this thesis of the mental logic theory can overcome that difficulty. As indicated, it is possible to assume that human reasoning follows logical rules without considering mental logic to be equivalent to Gentzen´s calculus, and, in this way, contradictions are not longer a problem. But this discussion is especially relevant here because step 4 in the deduction above ($\alpha \wedge \neg\alpha$) is a contradiction, and it can be thought that the fact that that contradiction is necessary for drawing $\alpha \to \beta$ from $\neg\alpha$ by itself explains why people tend not to accept the inference (even though it is logically valid).

However, the explanation given by the mental logic theory is really more detailed. Braine and O´Brien (1998b) explicitly study and analyze the problems of the inferences that have the logical structure $\neg\alpha \vdash \alpha \rightarrow \beta$ (Braine & O´Brien, 1986b, pp. 206-207). In their view, the application of the conditional introduction rule (that is, the rule used in step 7 of the previous derivation) has certain restrictions. In particular, those restrictions are three:

(R1) A contradiction only allows deducing that a previous assumption is false (this restriction has already been commented above).

(R2) A supposition α can become, by means of the use of the conditional introduction rule, the antecedent of a formula such as $\alpha \rightarrow \beta$ only if it is consistent with the previous assumptions and suppositions.

(R3) When α is supposed in order to apply the conditional introduction rule, no previous assumption contradicting α can be used until the rule has been completely applied.

In this way, if the deduction above is taken into account, it can be said that (R2) prohibits step 2, (R1) shows that step 5 is questionable, and (R3) prohibits step 4. Therefore, it is not difficult for the mental logic theory to explain why people do not often accept the paradoxical inferences with the form $\neg\alpha \vdash \alpha \rightarrow \beta$. Nevertheless, it is true that the mental logic theory admits the possibility of "a deliberately counterfactual supposition" (Braine & O´Brien, 1998b, p. 206), that is, the possibility that, as an exception to (R2), step 2 is allowed. The idea appears to be that, while (R1) and (R3) do not admit exceptions, individuals can think about fictional scenarios in which some aspect of reality is different. However, in those cases, the assumptions being true cannot be considered. For example, if we imagine a hypothetical scenario in which elephants can fly, we cannot consider, at the same time, the fact that elephants cannot really fly. This means, for the previous deduction, that, in the case that step 2 was a deliberately counterfactual supposition, step 1 could not be considered. Thus, the final result would be a deduction similar to this one:

(1) α (Supposition)
(2) β (Supposition)

Of course, step 2, instead of β, could be $\neg\beta$, but, in any case, it is clear that the deduction would not allow drawing $\alpha \rightarrow \beta$. So, following the mental logic theory, it is not surprising that people think that $\alpha \rightarrow \beta$ cannot be derived from $\neg\alpha$.

It is hence obvious that syntactic frameworks can explain the problem of this paradoxical inference as well. However, as said, Orenes and Johnson-Laird (2012) refer to versions of the inference $\neg\alpha \vdash \alpha \rightarrow \beta$ that individuals do accept, and, in their view, the mental models theory can also explain the latter fact and the syntactic theories cannot. The next section describes the characteristics of those versions and Orenes and Johnson-Laird´s (2012) explanation of why people accept them.

5. Modulation of Possibilities

The mental models theory also raises that modulation mechanisms can be used in order to block possibilities. Meanings and pragmatic and contextual factors can have an influence on models and, for this reason, some of them can be removed or ignored. Thus, according to Orenes and Johnson-Laird (2012), in order that individuals admit inferences with the structure $\neg\alpha \vdash \alpha \rightarrow \beta$, the semantic content needs to be manipulated. In particular, it is necessary that the content of β is equivalent to that of the premise. Based on the previous example, this can be made by means of the following conditional statement: "If it rains today then the forecast is wrong" (Orenes& Johnson-Laird, 2012, p. 363). The idea is to keep the premise (*It won´t rain today*), to eliminate the original conclusion, and to take this last conditional as a new conclusion. In this way, the key is that, in their opinion, what really means the new conclusion is: "If it rains today, which it won´t, then the forecast is wrong, which it isn´t" (Orenes & Johnson-Laird, 2012, p. 364). And this new conclusion, as it can be noted, does not refer to the model inconsistent with the premise, because it truly only allows one model:

$$\neg p \& \neg q$$

That is, it only allows this possibility:

It does not rain – The forecast is not wrong

Orenes and Johnson-Laird (2012), as indicated, carried out an experiment with the intent to check these theses and, indeed, their results seemed to demonstrate that they were right, since their participants tended to accept versions of the inference such as this one, which were called 'modulated versions' by them.

The question is now whether or not syntactic frameworks can also explain why people tend to admit the modulated versions. Orenes and Johnson-Laird (2012) think that they cannot give an account in this regard, but I do not agree. In my opinion, syntactic explanations of Orenes and Johnson-Laird´s (2012) experimental findings are possible. I will try to show it in the following section.

6. The Modulated Versions from a Syntactic Point of View

The key to understand what happens to the modulated versions from a syntactic point of view lies in the fact that the consequent of the conclusions of such versions refers to the premise. If the described Orenes and Johnson-Laird´s (2012) example is taken into account, it can be stated that it is not possible that it rains and, at the same time, the forecast is right. If it rains, the forecast is necessarily wrong, because, if the forecast is right, it must rain. Thus, if 'p' stands for 'it rains', it can be assumed that the real logical form of that example is $\neg p \vdash p \rightarrow p$ (or, if preferred, $\neg p \vdash p \rightarrow \neg\neg p$). The conclusion is $p \rightarrow p$, and not $p \rightarrow q$ because, as said, in this scenario, p and q seem to be equivalent, or at least so closely connected that it is possible to exchange one of them for the other. This can be seen more clearly if other examples of modulated inference given by Orenes and Johnson-Laird (2012) are taken into account. One of those examples is as follows: "The experiment won´t work. Does it follow that if the experiment works then this prediction is false?" (Orenes & Johnson-Laird, 2012, p. 374). Obviously, it can be thought that the logical form of the conclusion of this new inference is really $p \rightarrow p$, since the fact that the prediction is false means that the experiment works.

Something similar happens to this other example:

"The politicians won´t pass the budget. Does it follow that if the politicians do pass the budget then many of them changed their views?" (Orenes & Johnson-Laird, 2012, p. 374). Evidently, it this context, a change in politicians´ view involves that they pass the budget.

A fourth example is this one:

"Lorraine will win her tennis match. Does it follow that if Lorraine doesn´t win her tennis match then this expectation was incorrect?" (Orenes& Johnson-Laird, 2012, p. 374). Clearly, p and q are also equivalent here, since the expectation is that Lorraine wins the match.

One last example can be as follows:

"Lawrence is going on strike. Does it follow that if Lawrence doesn´t go on strike then his colleagues were mistaken about him?" (Orenes & Johnson-Laird, 2012, p. 374). It is also obvious in this case that what Lawrence´s colleagues thought about him is that he was going on strike and that, therefore, p and q can be exchanged in this scenario as well.. In my view, these analyses of Orenes and Johnson-Laird´s (2012) examples are strong enough to assume that the true logical form of the conclusions of their modulated inferences is not $p \rightarrow q$, but $p \rightarrow p$. However, if my argument is correct, it can be syntactically explained why the modulated versions tend to be considered as valid by people. As it can be checked in Table 2, $v(p \rightarrow p)$ = 1 both if $v(p)$ = 1 and if $v(p)$ = 0, which means that $p \rightarrow p$, unlike $p \rightarrow q$, is a tautology, and a tautology is hard to reject. Thus, it can be claimed that individuals usually accept the modulated versions of the inference $\neg\alpha \vdash \alpha \rightarrow \beta$ because the conclusions of such versions are really tautologies.

Table 2: Truth Table Corresponding to $p \rightarrow p$

p	$p \rightarrow p$
1	1
0	1

It is evident, nevertheless, that this argument can be questioned in a sense. If the cause that the paradoxical versions are not admitted is that their conclusions are less informative than their premises, the fact that the conclusion is a tautology does not seem to solve the problem. Tautologies are obvious sentences, and, precisely because they are obvious, it can be thought that they are uninformative. Nonetheless, this criticism can be responded in two ways.

Firstly, it can be argued that, as said, although tautologies are not very informative, they are difficult to reject under any circumstances. Thus, it can be stated that, in general, people do not accept inferences in which there is a premise that is more informative than the conclusion, and that, however, there is an exception for that. That exception refers to tautologies. Tautologies are unquestionable sentences and individuals tend to admit them in any scenario. But, if this argument is unconvincing, other account is possible too. The mental logic theory, which, as mentioned, speaks about a logic different to standard logic, can explain why individuals accept the modulated versions as well. In the scenario of the rain and the forecast, a fact is known: it will not rain, i.e., $\neg p$. Therefore, to suppose that it rains, i.e., to suppose p, is to raise "a deliberately counterfactual supposition" (Braine & O´Brien, 1998, p. 206). Nevertheless, as indicated, according to the mental logic theory, when a fictional scenario is imagined, actual data cannot be taken into account, which means that, if p is assumed, the premise $\neg p$ cannot be considered. In this way, the corresponding derivation would have this structure:

(1) p (Supposition)

It is evident that this only datum does not enable many more steps or many conclusions, but it is also clear that it does enable an interesting deduction. Step 2 can be p again by reiteration and, in this way, step 3 can be in turn $p \rightarrow p$ by the conditional introduction rule. The derivation hence would be as follows:

(1) p (Supposition)
(2) p (Reiteration 1)
(3) $p \rightarrow p$ ($\rightarrow$ Introduction 1-2)

None of these three steps contravenes (R1), (R2), or (R3). In addition, the *Reduction ad Absurdum* rule is not involved. Therefore, if it is assumed that the real logical form of the conclusions of the modulated versions of the inference $\neg\alpha \vdash \alpha \rightarrow \beta$ is not $\alpha \rightarrow \beta$, but $\alpha \rightarrow \alpha$, approaches such as that of the mental logic theory can also explain why such versions are accepted.

7. Conclusions

The previous pages show us that Orenes and Johnson-Laird´s (2012) experiment with paradoxical and modulated versions of the inference $\neg\alpha \vdash \alpha \rightarrow \beta$ is not decisive. Of course, it does not demonstrate that human mind works by means of syntactic rules (whether such rules are those of Gentzen´s propositional calculus or those of the mental logic theory). In fact, Orenes and Johnson-Laird´s (2012) explanation, which is based on the mental models theory, is very solid and convincing. However, the fact that their results can also be interpreted from syntactic frameworks can cause doubts and indicate to us that it is necessary to continue to research on these issues. Perhaps other kind of experiment that can be considered as more crucial is required, but we cannot forget that there is also the possibility to find links or relations between the semantic and the syntactic approaches. For example, López-Astorga (2013a, 2013b) raises that the same results can be interpreted both from the mental models theory and from a syntactic or formal theory, and arguments such as those given by Bermúdez (2003) or Heil (2009) enable us to think about an inclusive theory in which both logical rules and semantic models have a role. In any case, the real problem for the formal frameworks does not appear to be the contradictions or the *Reduction ad Absurdum* rule, because the difficulties that they generate can be eliminated by the mental logic theory.

Certainly, it is a problem that, in Gentzen´s propositional calculus, a contradiction can lead to any formula. Nonetheless, as indicated, following the mental logic theory, we must not assume all the rules and principles of that calculus. We can perfectly suppose that, in the real human reasoning, contradictions only allow affirming that an assumption is false. Furthermore, the *Reduction ad Absurdum* rule is not required to explain from syntax why the modulated versions of $\neg\alpha \vdash \alpha \rightarrow \beta$ tend to be considered as valid. The true problem is that it is hard to find a clear relation between logical forms and expressions in natural language. According to the mental models theory, to find such a relation is not needed, since human reasoning works analyzing possibilities or models, and, in its view, logical forms are not required. However, such forms are essential for the syntactic approaches, which require, if not an algorithm, at least a more or less systematic procedure, or a more or less rigorous method, for detecting them. Undoubtedly, this fact can be considered to be a plus for the mental models theory.

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